# Design of polarizing color filters with double-liquid-crystal cells

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A method of designing polarization rotators with double-liquid-crystal (LC) cells is presented. When placed between a polarizer and an analyzer, the polarization rotator becomes a polarizing color filter. Any required color can be generated by optimization of the parameters of the double-LC layers. One specific example of a green filter is given. This filter is analyzed in terms of the optical performance, including transmission spectrum, color coordinates, and viewing angle. A sample green polarizing color filter was made and compared with the theoretical results. © 2002 Optical Society of America

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## 1. Introduction

Polarization rotators (PRs) are devices that can rotate the linear polarization of a band of light by a fixed angle such as 90°. When placed between a polarizer and an analyzer, a PR becomes a polarizing color filter (PCF).[1] This type of color filter is interesting because the complementary color is not absorbed as in ordinary absorptive color filters.[2] Instead, it can be separated out and reused, if the analyzer is a polarizing beam splitter (PBS).[3] Thus the output of a PCF consists of two orthogonally propagating and orthogonally polarized complementary colors, such as green and magenta (G–M), or red and cyan (R–C), or blue and yellow (B–Y). Sharp *et al*. pointed out the application of such PCF in optical projection systems with reflective microdisplay light valves.[3] Ideally, red, green, and blue (RGB) primary colors can be separated from white light with a series of such PCFs.

As is well known in elementary optics, a single half-wave plate can be used as a 90° PR.[1] However, the transmission spectrum of such a PCF, which is given by $T = \cos^2 \pi d\Delta n/\lambda$, is neither broad enough nor narrow enough for actual applications. Here $d\Delta n$ is the retardation of the half-wave plate. A stack of many retardation plates can in principle be used to design a PR that has a much narrower bandwidth. Narrow-band color filters using this PCF approach have been designed and are variously known as Loyt filters and Solc filters depending on the retardation stack configuration.[1] The design of a PCF using a retardation stack is an interesting mathematical problem in itself, sometimes involving genetic algorithms.[4]

Besides retardation films, a liquid-crystal (LC) cell can also be used as a PR.[5] In fact, a 90° twisted-nematic cell in the waveguiding mode is an ideal broadband PR.[5] Much past research has been devoted to making a color supertwisted-nematic (STN) LCD using birefringent colors.[6] Such colors are based on the birefringent effect of the LC and in fact have requirements similar to those for PCF. In this paper we explore the use of double STN (DSTN) cells for making PCF for applications to projectors. In projectors one needs to separate white light into RGB primary colors. We demonstrate a design of green color PCF using DSTN. Quite satisfactory results can be obtained. One useful aspect of DSTN-based PCF is that the color can be tuned in principle by means of applying voltages to the LC cells to change their birefringence.

There are several potential advantages of using a LC cell instead of retardation films for making PCFs. The obvious one is that the number of optical elements can be greatly reduced. As shown previously, an LC cell is optically equivalent to three retardation plates.[7] In fact, a LC cell in general has three independent parameters, which is equivalent to the three orientation angles of three quarter-wave retardation films. Hence a DSTN arrangement is optically equivalent to six retardation films. As shown below, reasonably narrow color filters can be made with a DSTN arrange-

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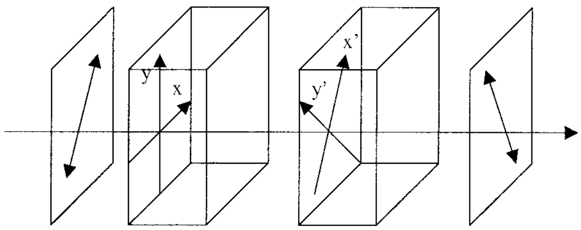


Fig. 1. Structure of the double-LC-layer color filter.

ment. Thus instead of laminating six retardation films, one needs only to make two LC cells.

The other advantage of using a LC cell is that once the LC cell is made, there is only one adjustable parameter (the rotation angle of the LC cell) in making the PCF, since the cell retardation and twist angle are fixed. Hence alignment and assembly can be quite straightforward. Finally, as pointed out earlier, LC cells can be tuned by means of applying a voltage,[8] whereas PCFs made with retardation films can apply only to one wavelength. However, we shall not be concerned with this aspect of the PCF in this paper.

## 2. Design of the Polarizing Color Filter

The sandwiched structure of analyzer–retarder–polarizer to function as a PCF has been studied for years. For the case of DSTN cells used as the retarders, the PCF structure is shown as Fig. 1. With respect to the $x$ axis, the analyzer and the polarizer had an input polarized angle $\alpha$ and an output polarized angle $\gamma$, the first and the second LC layers have twist angles of $\varphi_1$ and $\varphi_2$, and cell gaps of $d_1$ and $d_2$, respectively. If $\Delta n$ is the birefringence of the LC material, then the two cells can be characterized by retardations $\delta_1$ and $\delta_2$, where $\delta_i = \pi(d\Delta n)_i/\lambda$. Finally $\theta$ is the rotation angle from the input director of the first LC layer to that of the second one.

The LCD optical modeling is most conveniently performed by use of a Jones matrix to represent the polarization state of light passing through the system. The transmission of light normal incidence can be expressed as[9]

$$T = \left| (\cos\gamma \ \sin\gamma) R^{-1} M_{\mathrm{LC2}} R M_{\mathrm{LC1}} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \right|^2, \quad (1)$$

where $M_{\mathrm{LC}i}$ is the Jones matrix of the LC layer

$$M_{\mathrm{LC}i} = \begin{bmatrix} a - ib & -c - id \\ c - id & a + ib \end{bmatrix} \quad (2a)$$

where

$$a = \cos\varphi_i \cos\chi_i + (\varphi_i/\chi_i)\sin\varphi_i \sin\chi_i, \quad (2b)$$

$$b = (\delta_i/\chi_i)\cos\varphi_i \sin\chi_i \quad (2c)$$

$$c = \sin\varphi_i \cos\chi_i - (\varphi_i/\chi_i)\cos\varphi_i \sin\chi_i, \quad (2d)$$

$$d = \frac{\varphi_i}{\chi_i} \sin\varphi_i \sin\chi_i, \quad (2e)$$

$$\chi_i = (\varphi_i^{\,2} + \delta_i^{\,2})^{1/2}. \quad (3)$$

Table 1. Ranges of the Input Variables

| | $\alpha$ (deg) | $\gamma$ (deg) | $\varphi_i$ (deg) | $d_i$ ($\mu$m) | $\theta$ (deg) |
|---|---|---|---|---|---|
| From | −90 | −90 | −360 | 0 | 0 |
| To | 90 | 90 | 360 | 15 | 360 |

In Eq. (1), $R$ is the coordinate transformation matrix:

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \quad (4)$$

According to the above equations, the transmission of the DSTN cell is dependent on seven parameters: $\alpha$, $\gamma$, $\varphi_1$, $\varphi_2$, $d_1$, $d_2$, and $\theta$, in the zero voltage state. For any combination of ($\alpha$, $\varphi_1$, $d_1$, $\theta$, $\varphi_2$, $d_2$, $\gamma$) the transmission $T(\lambda)$ can be obtained as a function of wavelength $\lambda$.

In optimizing the PCF, we used a brute-force method of calculating the transmission spectra of all combinations of the seven variables. Essentially, we search the entire parameter space for combinations that give the desired transmission spectrum. In this calculation, the wavelength was changed from 0.4 to 0.7 $\mu$m in steps of 0.005 $\mu$m. The seven variables were varied such that there are 100 data points within a reasonable parameter range. For example, the twist angles $\varphi_1$ and $\varphi_2$ were varied from −360° to 360° in steps of 7.2°. With a fast computer the computation and search for the optimal filter design can be performed in a few hours.

In the particular example to be discussed in this paper we tried to design a green PCF. Therefore the criteria for a good filter are that (1) the transmission at a 0.54-$\mu$m wavelength is higher than 95% and (2) the transmission is less than 5% within the ranges of 0.42–0.47 $\mu$m and 0.62–0.7 $\mu$m. A computer program was designed to search for such solutions. Table 1 shows the ranges of the variables used. In the calculation the dependence of birefringence $\Delta n$ on wavelength is taken into account by use of the Cauchy equation[10]

$$\Delta n = A + (B/\lambda^2), \quad (5)$$

where $A$ and $B$ are the Cauchy parameters. For each set of variables that satisfied the design criteria, the color coordinates of the filter were calculated by the tristimulus distribution function given by the 1931 Commission Internationale de l'Éclairage (CIE) diagram.

## 3. Optical Analysis of the Green Polarizing Color Filter

### A. Transmission Spectra and Color Coordinates

Many optimized configurations were achieved by use of the above procedure. Two examples are shown in Table 2. The two LC layers of the first PCF (PCF1) had left-handed twist angles of 100° and 130°, and those of the second one (PCF2) had a right-handed twist angle of 140° and a left-handed twist angle of 40°. It should also be noted from Table 2 that the

Table 2. Values of the Seven Parameters for Two Good Designs

| | $\alpha$ (deg) | $\varphi_1$ (deg) | $d_1$ ($\mu$m) | $\theta$ (deg) | $\varphi_2$ (deg) | $d_2$ ($\mu$m) | $\gamma$ (deg) |
|---|---|---|---|---|---|---|---|
| CF1 | 70 | 130 | 8 | 350 | 100 | 8 | −70 |
| CF2 | −70 | −140 | 8 | 0 | 40 | 8 | 20 |

input and output polarizers are not necessarily parallel or perpendicular to each other. With these parameters the transmission spectra can be calculated as shown in Fig. 2. It can be seen that a relatively sharp and narrow transmission peak can be obtained for the green color.

As mentioned in the Section 1, the important feature of PCF is that both the filtered color and its complement can be obtained. For the case of the green filter, if the output analyzer angle is rotated by 90°, the complementary color magenta can be obtained. This is shown as Fig. 3. If an arrangement as shown in Fig. 4 is used, all the primary colors can be obtained. In Fig. 4 the magenta output from the first PBS is further separated by a second dichroic filter into blue and red. This dichroic filter is simple to implement, since there is a big gap in the middle of the magenta spectrum. This arrangement is useful for full-color projection displays.

It is thus helpful to calculate the RGB color coordinates of the color-separation system depicted in Fig. 4. The results are plotted in Fig. 5 where the 1931 CIE color coordinates are plotted for the three primary colors. The Society of Motion Picture and Television Engineers'-C standard for color television is included as a reference. From that figure, it can be seen that the color coordinates of the blue and the green are quite good. But the red is not quite saturated, mostly because of the green component present in the red channel. Presumably, a cleanup filter employing conventional thin-film coatings can be used to improve the color purity of the red channel.

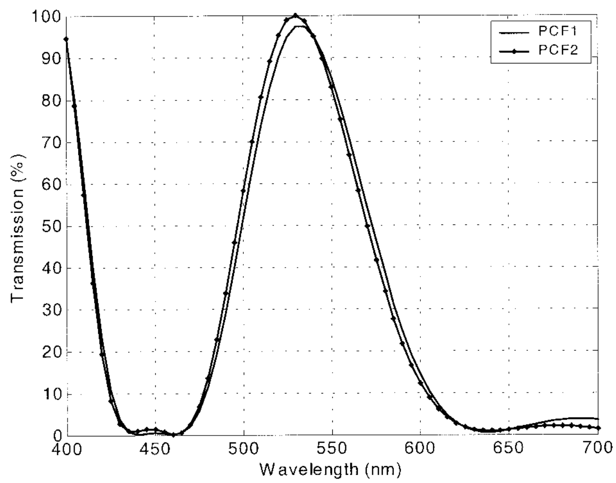


Fig. 2. Simulated transmission spectrum of the PCF1 and the PCF2.

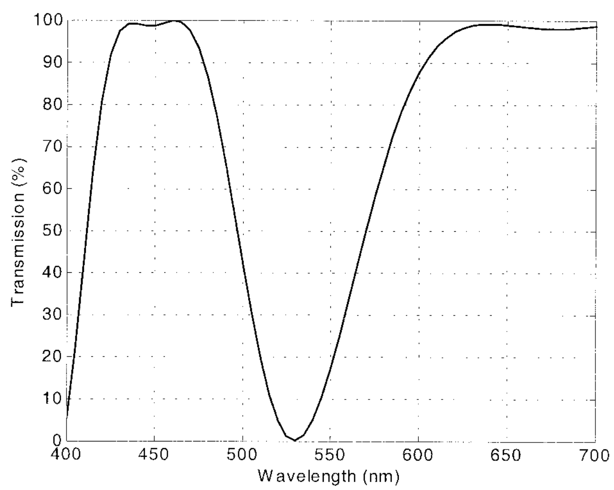


Fig. 3. Complementary transmission spectrum of the PCF1.

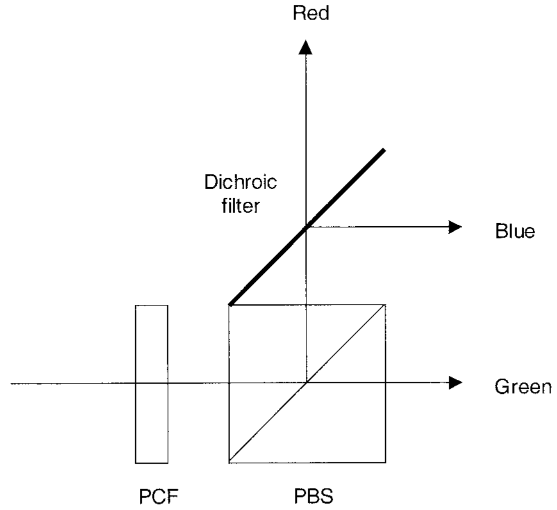


Fig. 4. Color separation system to obtain red, green, and blue.

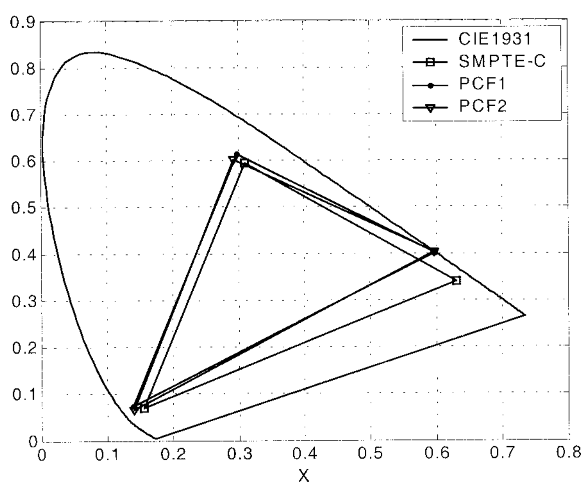


Fig. 5. Color coordinates of color separation systems using PCF1 and PCF2 as compared with the SMPTE-C standard.

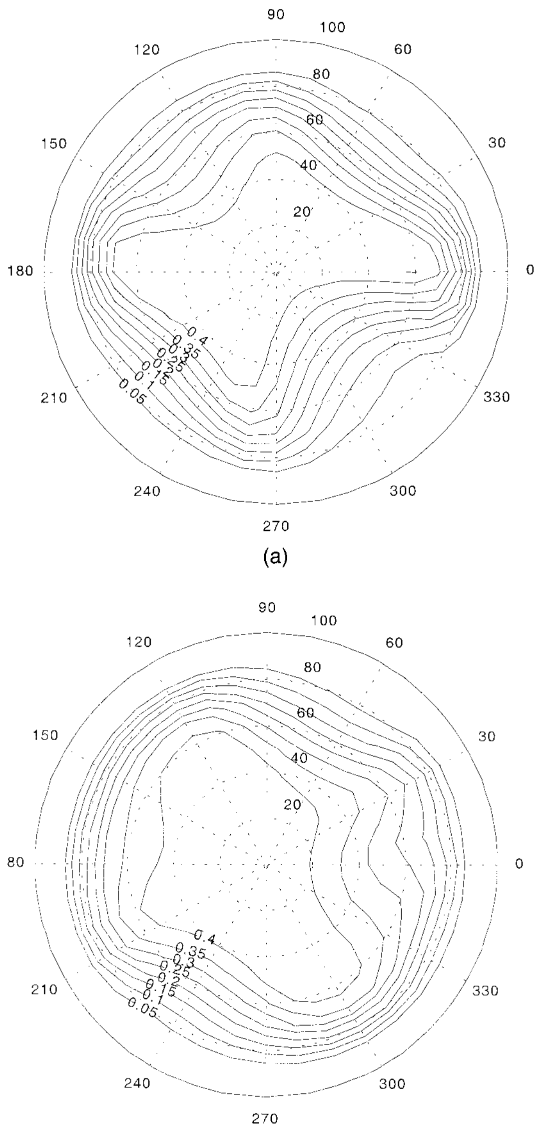


Fig. 6. Transmission contour of (a) PCF1 and (b) PCF2 with respect to the viewing angle.

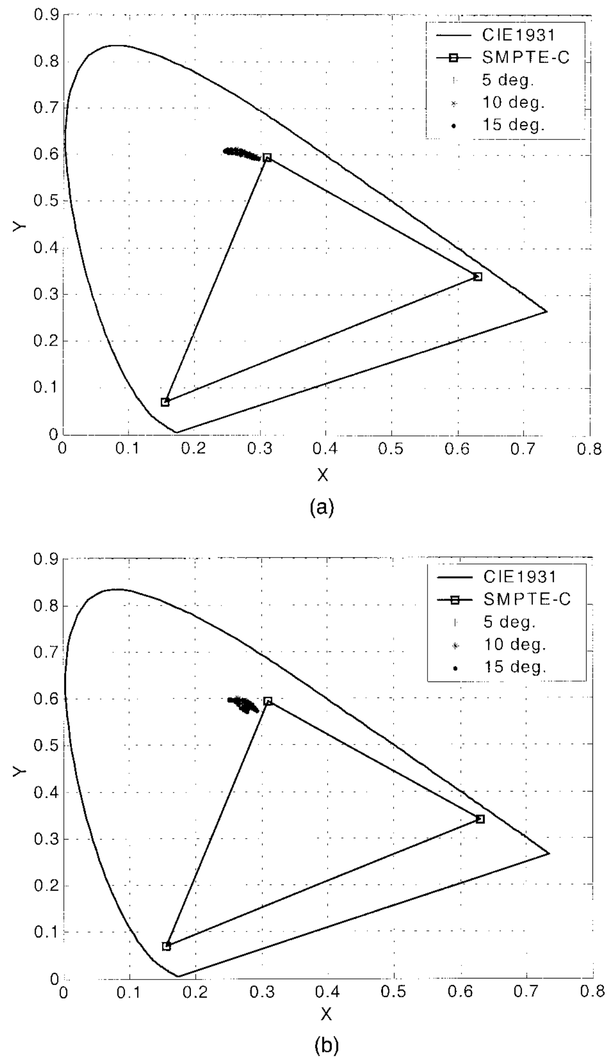


Fig. 7. Green color coordinates of (a) PCF1 and (b) PCF2. There are three sets of points for incident angles of 5°, 10°, and 15°.

### B. Viewing-Angle Effect

In general, the transmittance properties of a LC cell depend on the viewing direction, owing to the strong dependence of the birefringence on the viewing angle. To a PCF, the viewing angle may affect both the transmission and the color coordinate of the filter. This effect has to be studied carefully before the PCF can be useful.

The optical performance of the two PCF samples was simulated. Figures 6(a) and 6(b) show the contour plots of transmission of PCF1 and PCF2, respectively. In performing this calculation, we used our own software for calculating the optical properties of a LC cell at oblique angle of incidence.[11] The successive contours are in steps of 10%. The polar angle and the azimuthal angle were changed in steps of 10°. A transmission of >85% can be achieved in a cone of ±15° for both PCFs. This cone angle corresponds approximately to $F/2$ optics, which is more than sufficient to accommodate the divergence of the arc lamp light source. Hence these PCFs are useful as RGB color generators for projection displays employing such light sources.

In Fig. 7 we plot the color coordinates of each viewing direction within the cone of ±15° for the two PCFs. It can be seen that within this cone the color coordinates are all in a green color with varying degrees of saturation. Hence it is concluded that the green PCF remains green for most of the useful angular range. However, it is found that the color coordinates of PCF1 are generally better than those of PCF2. In fact, PCF1 produces green color in a cone of 25°, which might result from the high twist angles in this PCF design. Higher twist angle usually leads to a more symmetrical LC configuration, which results in a wider viewing-angle range.

## 4. Experimental Results

Two PCF samples were fabricated with the recipe given in the above discussions. The LC mixture

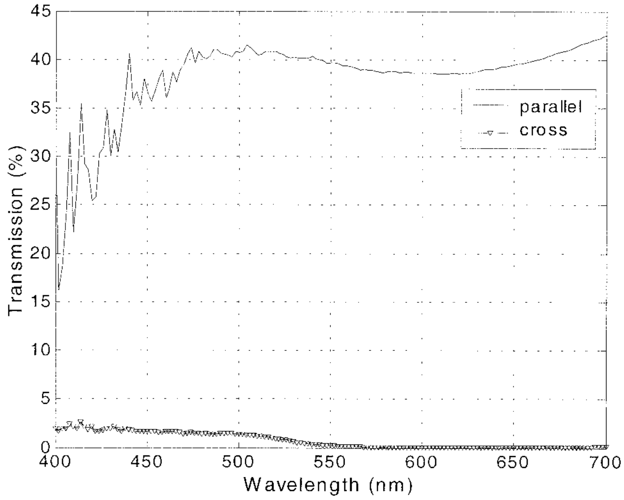


Fig. 8. Transmission of the polarizer and analyzer for parallel and cross geometry.

MLC-5700 was used for the LC layers in the experiments. The cells were made in a standard LCD fabrication line at the Center for Display Research.[12] The cell gap and birefringence of the LC were adjusted to be as close to the design values as possible. However, some discrepancy is unavoidable, since the cell gap cannot be adjusted continuously. For the polarizer and the analyzer, we used Nitto-Denko NPF-F1225DU and NPF-G1220DUN. They were chosen because of their high efficiency over the green color range. Unfortunately, polarizers that are both broadband and high efficiency are not available. They are especially poor in the blue color range and have significant light leakage there. Figure 8 shows the transmission of the polarizer and the analyzer under parallel and perpendicular alignment conditions. The maximum transmission is ~40%.

The double-LC-layer PCFs were illuminated by a collimated white-light source, and the optical properties are measured with a PR-705 spectrophotometer. The transmission spectra of the PCFs are shown in Fig. 9. Several observations can be made: (1) These curves agree well with the simulation results in terms of the shape of the transmittance curve. (2) The main difference is the lower peak transmission of the experimental results. Both PCF1 and PCF2 have a peak transmittance of ~30%. Given that the polarizer–analyzer efficiency is ~40%, this implies that the PCF has an efficiency of ~75%. This is somewhat lower than expected. It is believed that the major source of error is the thickness of the LC cells, which cannot be made precisely to the specified values.

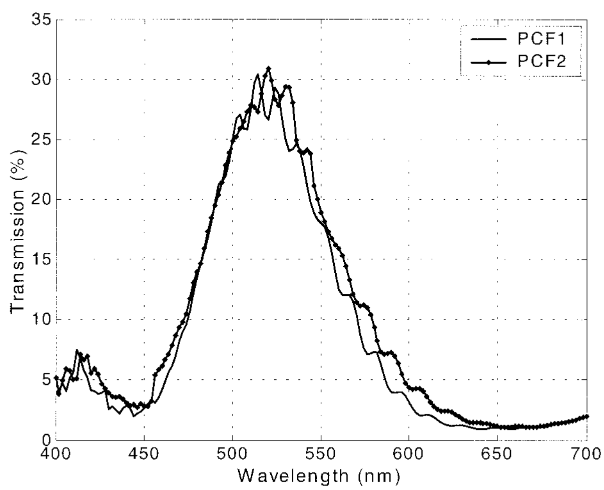


Fig. 9. Experimental transmission spectrum of PCF1 and PCF2.

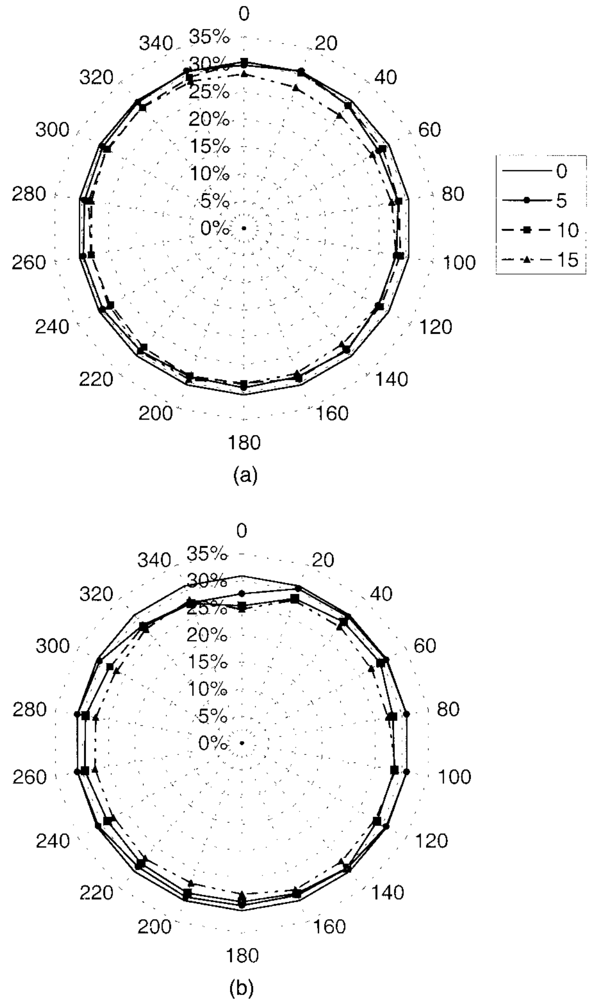


Fig. 10. Measured transmission as a function of polar angle for various azimuthal angles for (a) PCF1 and (b) PCF2.

The viewing-angle effect of these PCFs was also measured. Experimentally, the peak transmission of the PCF was measured as a function of $\varphi$ for $\theta = 5°$, 10°, and 15°. The results are shown in Fig. 10. In that diagram we plot the transmission as a function of the polar angle for the three $\theta$ values. They are in rough agreement with the simulation results. PCF1 has symmetrical transmittance as a function of the polar angle. Again the results show that these PCFs are suitable for applications in color separation.

For each set of values of $\varphi$ and $\theta$ we also measured

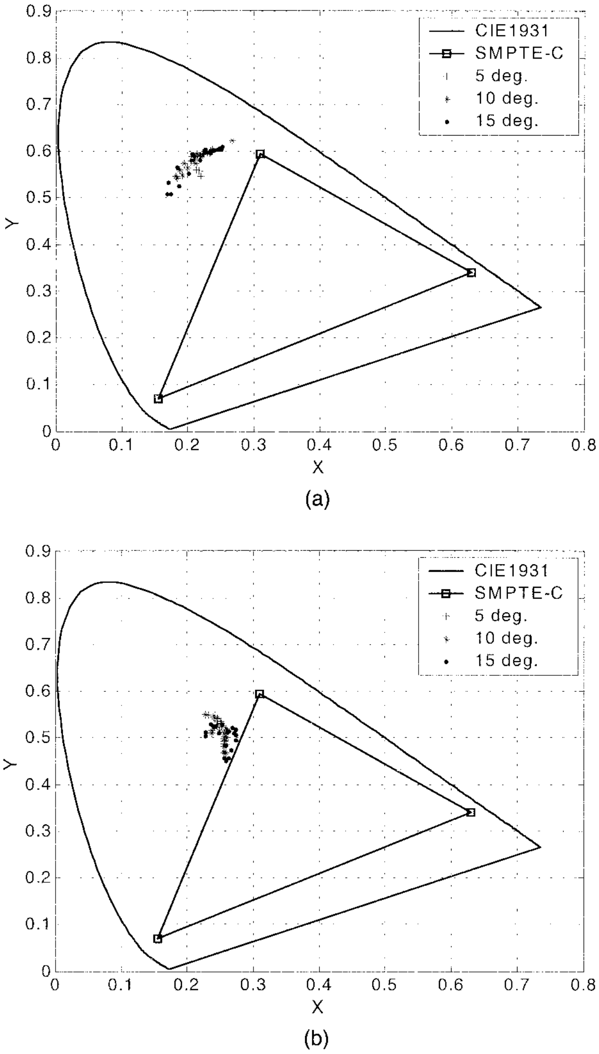


Fig. 11. Measured color coordinates of (a) PCF1 and (b) PCF2 for incident angles of 5°, 10°, and 15°.

the color coordinates, using the PR650. Figure 11 shows the color coordinates for transmitted light for $\theta = 5°$, 10°, and 15°. The polar angle was changed in steps of 20° in these experiments. It can be seen that the results are in fair agreement with the simulation results. However, the measured color coordinates tend to be toward the blue–green region rather than true green for most of the viewing angles. There may be two reasons for this color shift: (1) The design value of $d\Delta n$ is probably not exactly duplicated in the experiments. (2) The polarizer and analyzers have leakage in the blue, as evidenced in the transmission spectra in Figs. 8 and 9. This blue color leakage pulls the color coordinates toward the blue. Obviously these problems need to be addressed in further improvements of the PCF. In general, however, PCF1 is better than PCF2.

## 5. Conclusion

In this paper we have presented the general design of polarizing color filters (PCFs), using a double-LC-cell configuration. A complete search of the seven-dimensional parameter space resulted in many possible configurations for the green PCF. We concentrated on two of the designs and obtained the transmission spectra as well as the viewing-angle dependence of these filters. In particular we showed the viewing-angle dependence of the color coordinates of these filter designs and concluded that they were suitable for application to projection displays requiring $F/2$ optics. One merit of the PCF is that both the desired color and its complement could be obtained with a PBS. We presented a possible arrangement whereby the PCF can be used to obtain all primary colors of red, green, and blue.

Sample PCFs are made according to the design parameters. The optical properties of these experimental samples were measured. It was found that the experimental data generally agreed well with the simulation results, which proved that the design method of the PCF was valid. Both the simulation and the experiments showed that the polarization interference green filter had a good color quality over a large incident angle. It is suitable for $F/2$ optics. In the future other types of PCF at other wavelengths should be tested with the double-LC-cell concept.

From the experimental results we noted two problems: (1) The peak transmission efficiency of the PCF is not 100%. (2) The color tends to shift toward the blue. Both problems are in fact related to the quality of the polarizer–analyzer. We need polarizers that are both broadband and have good transmission efficiency. Unfortunately commercial polarizers are always a trade-off between good efficiency or bandwidth. Good-efficiency polarizers always have significant leakage in the blue. Broadband polarizers are not efficient. Presumably, if PBSs are used, the results will be better.

Finally, it should be noted that we have studied only the quiescent operation of the PCF in this paper. Here we emphasize the application to a projector in color separation. For the LC cell the birefringence can be tuned electrically. Thus the transmission spectrum of the PCF should be tunable. However, predicting the tuning behavior of such PCF requires much more extensive analysis and falls beyond the scope of the current paper.

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### References

1. P. Yeh, *Optical Waves in Layered Media* (Wiley, New York, 1988).
2. R. W. Sabnis, "Color filter technology for liquid crystal displays," Displays **20,** 119–129 (1999).
3. G. D. Sharp and J. R. Birge, "Retarder stack technology for color manipulation," in *Society for Information Display Symposium Digest* (Society for Information Display, San Jose, 1999), pp. 1072–1075.
4. X. J. Yu, Y. B. He, J. Y. Xiong, F. H. Yu, and H. S. Kwok, "Design of a polarization interference filter," Displays **23,** 145–149 (2002).
5. W. C. Yip, H. C. Huang, and H. S. Kwok, "Efficient polarization

converter for projection displays," Appl. Opt. **36,** 6453–6457 (1997).
6. K. H. Kang, J. M. Moon, T.-H. Yoon, J. C. Kim, G.-D. Lee, and E. S. Lee, "Design of a reflective multicolor STN-LCD on the Poincare sphere," in *International Display Research Conference Proceedings* (Euro Display), (IDRC, Birmingham, 1996), pp. 383–386.
7. S. T. Tang and H. S. Kwok, "$3 \times 3$ matrix for unitary optical systems," J. Opt. Soc. Am. A **18,** 2138–2145 (2001).
8. K. Vermeirsch and B. Maximus, "Design method for a fast switching color filter based on ECB cells," Mol. Cryst. Liq. Cryst. **351,** 79–86 (2000).
9. H. S. Kwok, "Parameter space representation of liquid crystal display operating modes," J. Appl. Phys. **80,** 3687–3693 (1996).
10. I. C. Khoo and S. T. Wu, *Optics and Nonlinear Optics of Liquid Crystals* (World Scientific, Singapore, 1993).
11. F. H. Yu and H. S. Kwok, "Comparison of extended Jones matrices for twisted nematic liquid-crystal displays at oblique angles of incidence," J. Opt. Soc. Am. A **16,** 2772–2780 (1999).
12. See the following URL: http//www.cdr.ust.hk.